

Variational characterization of norm convergence of alternating projection method

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The alternating projection method is one of the simplest method to find a solution for the convex feasibility problem for two nonempty closed convex subsets A and B of a Hilbert space \mathcal{H} , i.e., the problem to find a point belonging to $A \cap B$. The sequence generated by the alternating projection method is always weak convergent ([3]), whereas the celebrated example by Hundal ([5]) shows that the norm convergence is not always guaranteed. One of the most known sufficient conditions to obtain the norm convergence of the alternating projection method to some point of the best approximation set is the notion of regular pair (A, B) for two nonempty closed convex subsets A and B of a Hilbert space \mathcal{H} introduced by Borwein and Bauschke in 1993 ([2]).

The aim of this talk is to show that the regularity implies much more than the norm convergence of the alternating projection method. Indeed, we prove that the regularity of the pair (A, B) is equivalent to the convergence for any variational perturbation of the alternating projection method, provided the corresponding best approximation sets are bounded. Namely, let us consider two sequences of closed convex sets $\{A_n\}$ and $\{B_n\}$ converging with respect to the Attouch-Wets convergence to A and B , respectively. Given a starting point a_0 , we consider the sequences of points obtained by projecting onto the “perturbed” sets, i.e., the sequences $\{a_n\}$ and $\{b_n\}$ defined inductively by $b_n = P_{B_n}(a_{n-1})$ and $a_n = P_{A_n}(b_n)$. Suppose that $A \cap B$ is bounded, we prove that the couple (A, B) is regular if and only the couple (A, B) is d -stable, i.e., for each $\{a_n\}$ and $\{b_n\}$ as above we have $\text{dist}(a_n, A \cap B) \rightarrow 0$ and $\text{dist}(b_n, A \cap B) \rightarrow 0$. Similar results are obtained also in the case $A \cap B = \emptyset$, considering the set of best approximation pairs instead of $A \cap B$.

This talk is based on the papers [4] (with C.A. De Bernardi) and [1] (with F. Battistoni, A. Daniilidis, C.A. De Bernardi).

References

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- [4] C .A. De Bernardi and E. Miglierina, *Regularity and stability for a convex feasibility problem* Set-Valued Var. Anal. **30** (2022), 521–542.
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