COMPITO ELETTROTECNICA 07/03/2013

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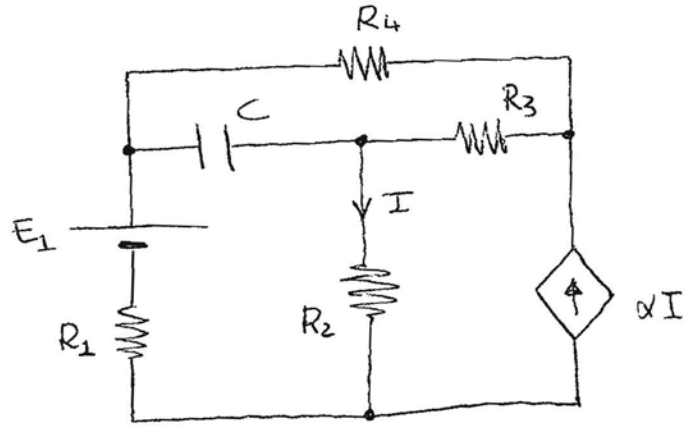
Corso di Laurea: ______________________________________

Esercizio 1:

Il circuito in figura è a regime. Determinare l'energia immagazzinata nel condensatore C.

$E_1 = 5$V; $R_1= 2\Omega$; $R_2= 5\Omega$; $R_3= 3\Omega$; $R_4= 1\Omega$; $\alpha=2$, C=1mF

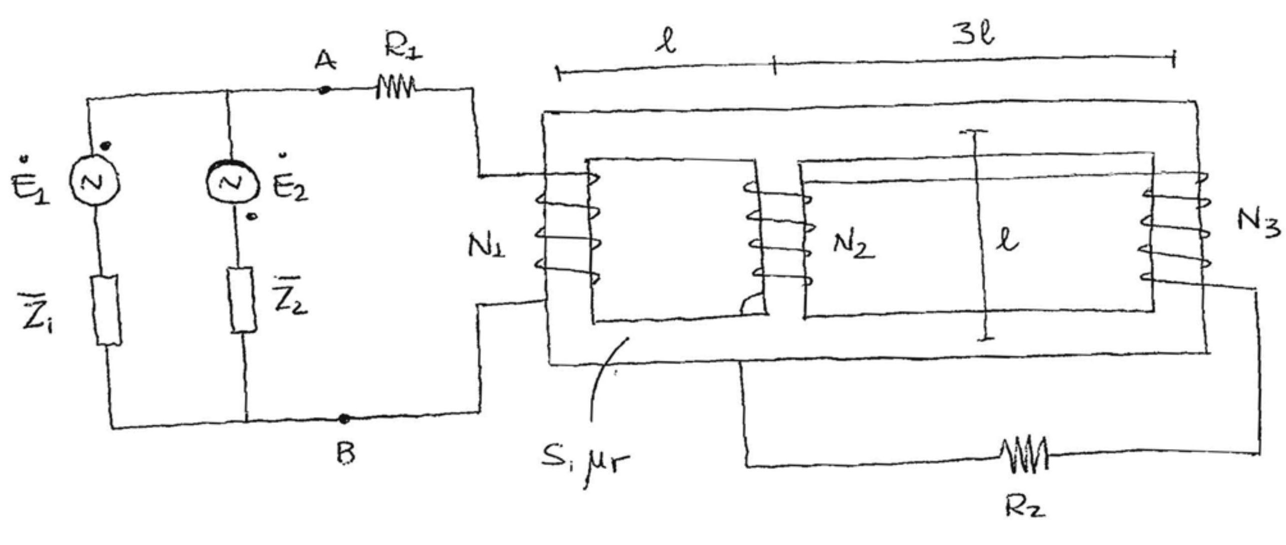


Esercizio 2:

Dato il seguente circuito a regime determinare la capacità **C** da inserire tra i punti A e B atta a rifasare il sistema a cosφ=0.7.

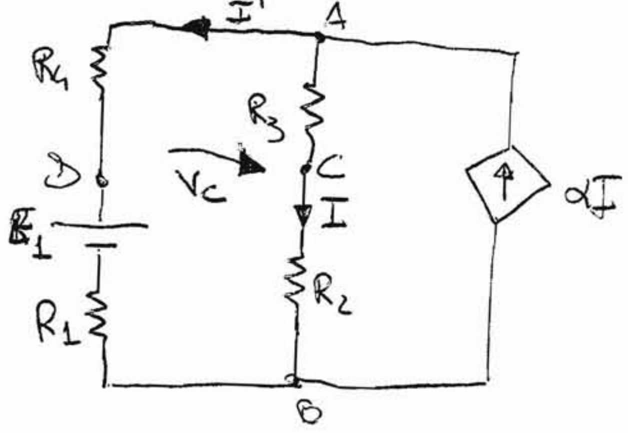
$R_1=2\Omega$; $R_2=6\Omega$; f=50Hz, $N_1=100$, $N_2=120$, $N_3=130$, l=10cm, S=10cm^2, $\mu_r=1000$,

$\dot{E}_2 = 4e^{j\omega t}$ V; $\dot{E}_1 = 5e^{j\left(\omega t+\frac{\pi}{3}\right)}$V $\bar{Z}_1$=5+1j [Ω]; $\bar{Z}_2$=3+2j [Ω]

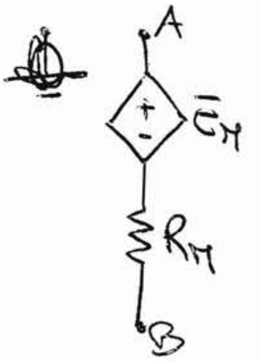


Esercizio 1

Il circuito si trova a regime, di conseguenza il condensatore si comporta da circuito aperto.



Applichiamo Millman a destra e tra i morsetti:



$$E_M = \frac{E_1/(R_1+R_4) + \alpha I}{\frac{1}{R_1+R_4} + \frac{1}{R_2+R_3}} = \frac{\frac{5}{3} + 2I}{\frac{1}{3} + \frac{1}{8}} = (3{,}63 + 4{,}36I)\,V$$

$$R_M = \frac{1}{\frac{1}{R_1+R_4} + \frac{1}{R_2+R_3}} = \frac{1}{\frac{1}{3} + \frac{1}{8}} = 2{,}18\ \Omega$$

Guardando il primo circuito, la V_{AB}, passando per il ramo in cui scorre I, è uguale:

$$V_{AB} = (R_3+R_2)I \Rightarrow V_{AB} = E_M \Rightarrow E_M = (R_3+R_2)I$$

$$3{,}63 + 4{,}36I = 8I \Rightarrow I = 1\,A$$

Continuando a considerare il primo circuito, la tensione ai capi del condensatore è:

$$V_C = V_{CA} + V_{AD} = -R_3 I + R_4 I'$$

Ricaviamo la I' dalla legge al nodo A:

$$\alpha I - I - I' = 0 \Rightarrow I' = \alpha I - I = 1\,A$$

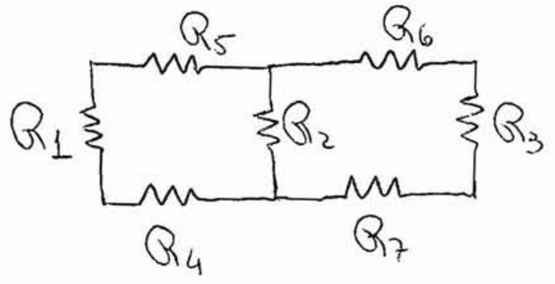
$$V_C = -3 + 1 = -2\,V$$

L'energia immagazzinata nel condensatore è:

$$W_C = \frac{1}{2} C V_C^2 = \frac{1}{2} \cdot (10^{-3}) \cdot (-2)^2 = 2 \cdot 10^{-3}\,J = 2\,mJ$$

Esercizio 2

Risolviamo il nucleo magnetico



$$\mathcal{R}_1 = \mathcal{R}_2 = \mathcal{R}_3 = \mathcal{R}_4 = \mathcal{R}_5 = \frac{l}{\mu \cdot \mu_r \cdot S} = \frac{10 \cdot 10^{-2}}{4\pi \cdot 10^{-7} \cdot 10^{3} \cdot 10 \cdot 10^{-4}} = 7{,}96 \cdot 10^4 \ H^{-1}$$

$$\mathcal{R}_6 = \mathcal{R}_7 = \frac{3\, l}{\mu \cdot \mu_r \cdot S} = 23{,}88 \cdot 10^4 \ H^{-1}$$

Calcoliamo la riluttanza equivalente vista dalla bobina 1:

$$\mathcal{R}_S = \mathcal{R}_6 + \mathcal{R}_7 + \mathcal{R}_3 = 2 \cdot 23{,}88 \cdot 10^4 + 7{,}96 \cdot 10^4 = 55{,}72 \cdot 10^4 \ H^{-1}$$

$$\mathcal{R}_P = \frac{\mathcal{R}_S \cdot \mathcal{R}_2}{\mathcal{R}_S + \mathcal{R}_2} = \frac{55{,}72 \cdot 10^4 \cdot 7{,}96 \cdot 10^4}{55{,}72 \cdot 10^4 + 7{,}96 \cdot 10^4} = 6{,}97 \cdot 10^4 \ H^{-1}$$

$$\mathcal{R}_{eq_1} = \mathcal{R}_P + \mathcal{R}_5 + \mathcal{R}_4 + \mathcal{R}_1 = 6{,}97 \cdot 10^4 + 3\left(7{,}96 \cdot 10^4\right) = 30{,}85 \cdot 10^4 \ H^{-1}$$

Calcoliamo la riluttanza equivalente vista dalla bobina 2:

$$\mathcal{R}_S = \mathcal{R}_3 + \mathcal{R}_6 + \mathcal{R}_7 = 55{,}72 \cdot 10^4 \ H^{-1}$$

$$\mathcal{R}_{S_2} = \mathcal{R}_1 + \mathcal{R}_4 + \mathcal{R}_5 = 3 \cdot 7{,}96 \cdot 10^4 = 23{,}88 \cdot 10^4 \ H^{-1}$$

$$\mathcal{R}_{P_2} = \frac{1}{\frac{1}{\mathcal{R}_S} + \frac{1}{\mathcal{R}_{S_2}}} = \frac{1}{\frac{1}{55{,}72 \cdot 10^4} + \frac{1}{23{,}88 \cdot 10^4}} = 16{,}72 \cdot 10^4 \ H^{-1}$$

$$\mathcal{R}_{eq_2} = \mathcal{R}_{P_2} + \mathcal{R}_2 = 16{,}72 \cdot 10^4 + 7{,}96 \cdot 10^4 = 24{,}68 \cdot 10^4 \ H^{-1}$$

Calcoliamo la riluttanza equivalente vista dalla bobina 3:

$$\mathcal{R}_{S_2} = \mathcal{R}_1 + \mathcal{R}_4 + \mathcal{R}_5 = 23,88 \cdot 10^4 \ H^{-1}$$

$$\mathcal{R}_{P_3} = \frac{\mathcal{R}_2 \cdot \mathcal{R}_{S_2}}{\mathcal{R}_2 + \mathcal{R}_{S_2}} = \frac{7,96 \cdot 10^4 \cdot 23,88 \cdot 10^4}{7,96 \cdot 10^4 + 23,88 \cdot 10^4} = 5,97 \cdot 10^4 \ H^{-1}$$

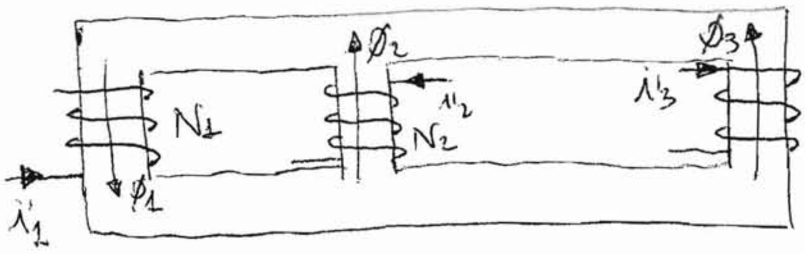
$$\mathcal{R}_{eq_3} = \mathcal{R}_6 + \mathcal{R}_7 + \mathcal{R}_{P_2} + \mathcal{R}_3 = 2 \cdot 23,88 \cdot 10^4 + 5,97 \cdot 10^4 + 7,96 \cdot 10^4 = 61,69 \cdot 10^4 \ H^{-1}$$

$$L_1 = \frac{N_1^2}{\mathcal{R}_{eq_1}} = \frac{10^4}{30,85 \cdot 10^4} = 32,4 \ mH$$

$$L_2 = \frac{N_2^2}{\mathcal{R}_{eq_2}} = \frac{120^2}{24,68 \cdot 10^4} = 58,3 \ mH$$

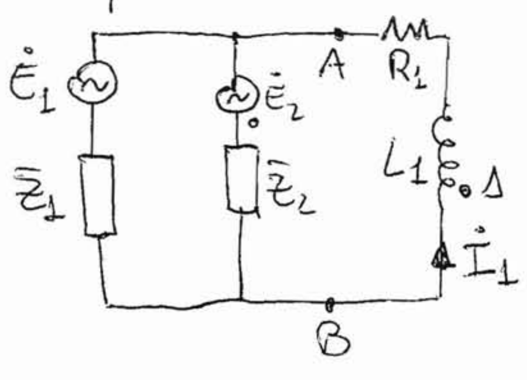
$$L_3 = \frac{N_3^2}{\mathcal{R}_{eq_3}} = \frac{130^2}{61,69 \cdot 10^4} = 27,4 \ mH$$

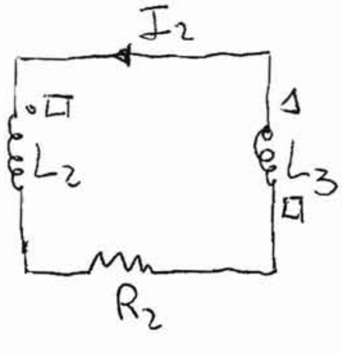
Stabiliamo il segno delle mutue



$\Rightarrow$ Equivalente elettrico del nucleo.

Equivalente elettrico del circuito





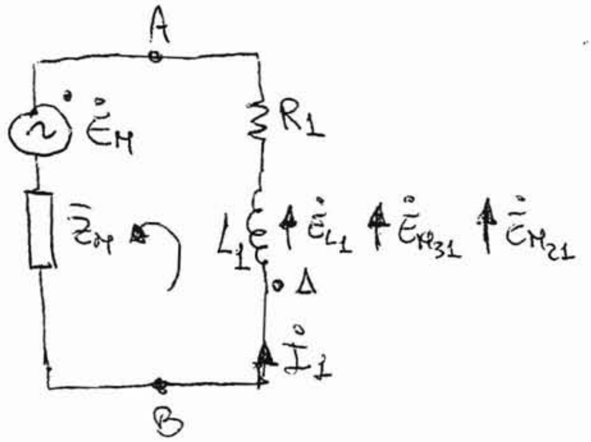
$$\dot{E}_1 = 5e^{j\left(\omega t+\frac{\pi}{3}\right)} = 5\left(\cos\frac{\pi}{3} + j\sin\frac{\pi}{3}\right) = 2{,}5 + j4{,}33\ V$$

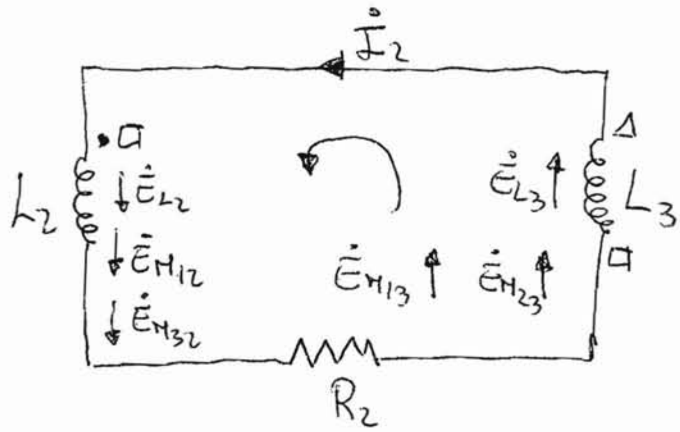
$$\dot{E}_2 = 4\ V$$

Applichiamo Millman tra i rami $\dot{E}_1 - \bar{Z}_1$ e $\dot{E}_2 - \bar{Z}_2$:

$$\dot{E}_M = \frac{\frac{\dot{E}_1}{\bar{Z}_1} - \frac{\dot{E}_2}{\bar{Z}_2}}{\frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2}} = \frac{\frac{2{,}5+j4{,}33}{5+j} - \frac{4}{3+j2}}{\frac{1}{5+j} + \frac{1}{3+j2}} = -1{,}74 + j2{,}40\ V$$

$$\bar{Z}_M = \frac{1}{\frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2}} = \frac{1}{\frac{1}{5+j} + \frac{1}{3+j2}} = 1{,}96 + j0{,}89\ \Omega$$





Coefficienti di ripartizione dei flussi:

$$\alpha_{12} = \frac{\mathcal{R}_3 + \mathcal{R}_6 + \mathcal{R}_7}{\mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_6 + \mathcal{R}_7} = \frac{7{,}96\cdot 10^4 + 2\cdot 23{,}88\cdot 10^4}{2\cdot 7{,}96\cdot 10^4 + 2\cdot 23{,}88\cdot 10^4} = 0{,}875$$

$$\alpha_{13} = \frac{\mathcal{R}_2}{\mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_6 + \mathcal{R}_7} = \frac{7{,}96\cdot 10^4}{2\cdot 7{,}96\cdot 10^4 + 2\cdot 23{,}88\cdot 10^4} = 0{,}125$$

$$\alpha_{23} = \frac{\mathcal{R}_1 + \mathcal{R}_3 + \mathcal{R}_4}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_4 + \mathcal{R}_3 + \mathcal{R}_6 + \mathcal{R}_7} = \frac{3\cdot 7{,}96\cdot 10^4}{4\cdot 7{,}96\cdot 10^4 + 2\cdot 23{,}88\cdot 10^4} = 0{,}300$$

Calcoliamo i coefficienti di mutua:

$$M_{12} = M_{21} = \frac{N_1 \cdot N_2}{Req_1} \alpha_{12} = \frac{12 \cdot 10^3}{30{,}85 \cdot 10^4} 0{,}875 = 34{,}0 \text{ mH}$$

$$M_{13} = M_{31} = \frac{N_1 \cdot N_3}{Req_1} \cdot \alpha_{13} = \frac{13 \cdot 10^3}{30{,}85 \cdot 10^4} 0{,}125 = 5{,}3 \text{ mH}$$

$$M_{23} = M_{32} = \frac{N_2 \cdot N_3}{Req_2} = \frac{15{,}6 \cdot 10^3}{24{,}68 \cdot 10^4} 0{,}300 = 19\,\text{mH}$$

Scriviamo le equazioni alle due maglie:

$$\begin{cases} -\dot{E}_M + \dot{E}_{L1} + \dot{E}_{M_{31}} + \dot{E}_{M_{21}} = (R_1 + j\omega L_1 + \dot{Z}_M)\dot{I}_1 \\ \dot{E}_{L_2} + \dot{E}_{M_{12}} + \dot{E}_{M_{32}} + \dot{E}_{L_3} + \dot{E}_{M_{13}} + \dot{E}_{M_{23}} = R_2 \dot{I}_2 \end{cases}$$

$$\begin{cases} -\dot{E}_M - j\omega L_1 \dot{I}_1 + j\omega M_{31} \dot{I}_2 - j\omega M_{21} \dot{I}_2 = 3{,}96\, \dot{I}_1 + j\,11{,}07\, \dot{I}_1 \\ -j\omega L_2 \dot{I}_2 - j\omega M_{12} \dot{I}_1 - j\omega M_{32} \dot{I}_2 - j\omega L_3 \dot{I}_2 + j\omega M_{13} \dot{I}_1 - j\omega M_{23} \dot{I}_2 = 6 \dot{I}_2 \end{cases}$$

$$\begin{cases} +1{,}74 - j2{,}40 - j10{,}18\, \dot{I}_1 + j1{,}67\, \dot{I}_2 - j10{,}68\, \dot{I}_2 = (3{,}96 + j11{,}07)\dot{I}_1 \\ -j18{,}31\, \dot{I}_2 - j10{,}68\, \dot{I}_1 - j5{,}97\, \dot{I}_2 - j8{,}61\, \dot{I}_2 + j1{,}67\, \dot{I}_1 - j5{,}97\, \dot{I}_2 = 6 \dot{I}_2 \end{cases}$$

$$\begin{cases} 1{,}74 - j2{,}40 - (3{,}96 + j21{,}25) - j9{,}01\, \dot{I}_2 = 0 \\ -(6 + j38{,}86)\dot{I}_2 - j9{,}01\, \dot{I}_1 = 0 \end{cases}$$

$$\begin{cases} \dot{I}_2 = -0{,}27 - j0{,}19 + (-2{,}36 + j0{,}44)\dot{I}_1 \\ -5{,}76 + j11{,}63 + (31{,}26 + j89{,}07)\dot{I}_1 - j9{,}01\, \dot{I}_1 = 0 \end{cases}$$

$$\begin{cases} \dot{I}_2 = 0{,}01 + j0{,}03 \text{ A} \\ \dot{I}_1 = \dfrac{-5{,}76 + j11{,}63}{-31{,}26 - j80{,}06} = -0{,}10 - j0{,}11 \text{ A} \end{cases}$$

$$\dot{V}_{BA} = -\dot{E}_M - \bar{Z}_M \dot{I}_1 \Rightarrow \dot{V}_{BA} = 1,74 - j2,40 - (1,96 + j0,89)(-0,10 - j0,12) =$$

$$= 1,84 - j2,10 \text{ V}$$

$$\bar{S}_{BA} = \dot{V}_{BA} \cdot \overset{\vee}{I}_1 = (1,84 - j2,10)(-0,10 + j0,11) = 0,05 + j0,41 \text{ VAC}$$

$$\phi_{CA} = \operatorname{arctg} \frac{Q_{CA}}{P_{CA}} = \operatorname{arctg} \frac{0,41}{0,05} = 83^\circ \Rightarrow \cos\phi_{CA} = 0,12$$

$$\cos\phi_2 = 0,7 \Rightarrow \phi_2 = \arccos 0,7 = 45,6^\circ \Rightarrow tg\,\phi_2 = 1,02$$

$$tg\,\phi_2 = \frac{Q_{CA} - Q_C}{P_{CA}} = \frac{Q_{CA} - \omega C V_{BA}^2}{P_{CA}}$$

$$V_{BA}^2 = (1,84)^2 + (-2,10)^2 = 7,8 \text{ V}^2$$

La capacità C atta a rifasare il carico a $\cos\phi_2 = 0,7$ è:

$$C = \frac{Q_{CA} - P_{CA}\, tg\,\phi_2}{\omega V_{BA}^2} = \frac{0,41 - 0,05 \cdot 1,02}{314,16 \cdot 7,8} = 146,5 \text{ µF}$$