COMPITO ELETTROTECNICA 04/07/2012

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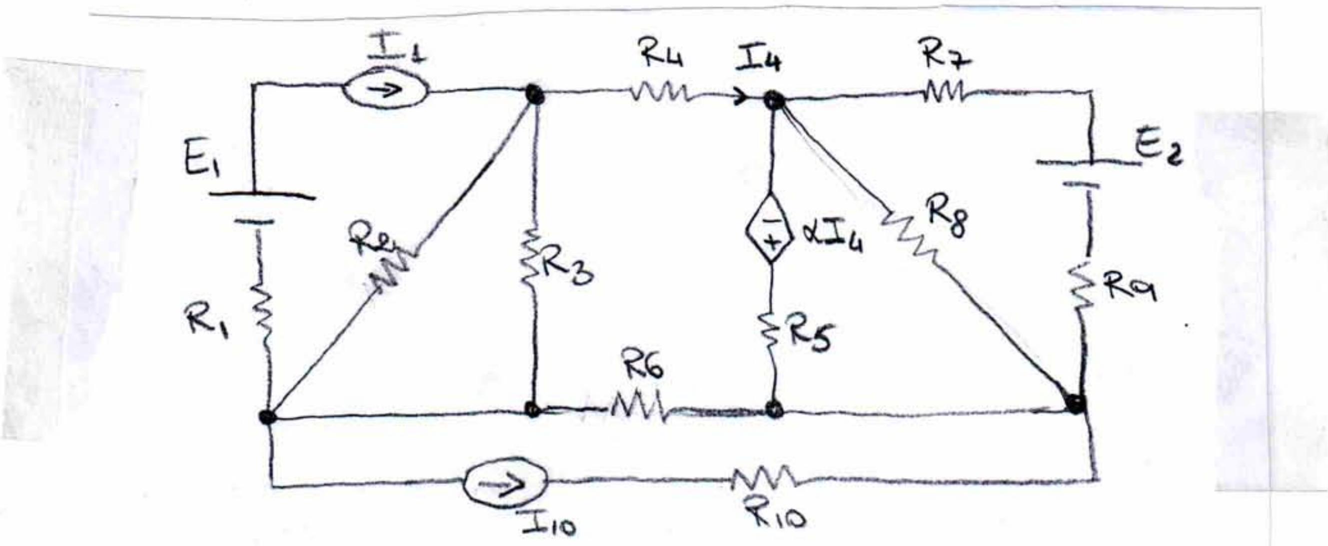
Corso di Laurea: ____________________________________

Esercizio 1:

Determinare i valori della potenza generata ed erogata su E_1 e E_2.

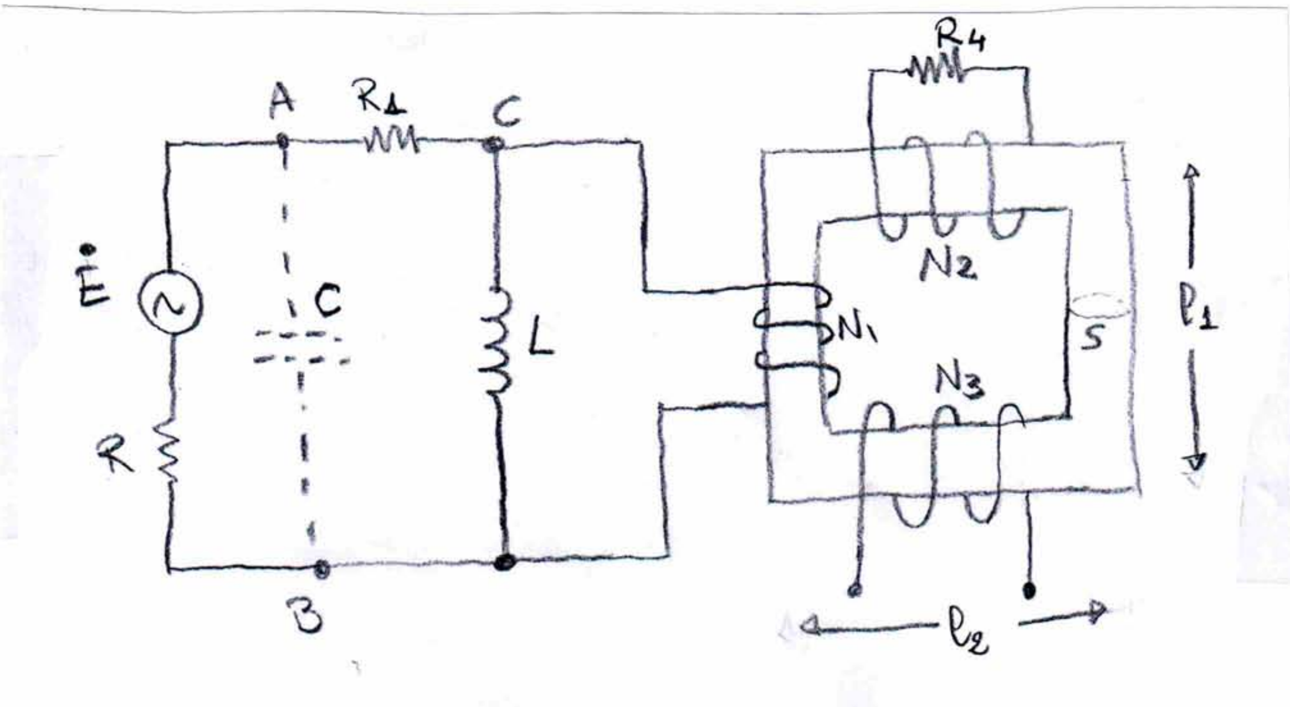
$E_1 = 5V$; $E_2 = 2V$; $R_1 = 2\Omega$ (resistenza interna di E_1); $R_2 = 1\Omega$; $R_3 = 2\Omega$; $R_4 = 5\Omega$; $R_5 = 3\Omega$; $R_6 = 4\Omega$; $R_7 = 2\Omega$; $R_8 = 6\Omega$; $R_9 = 10\Omega$ (resistenza interna di E_2); $\alpha = 2\Omega$; $I_1 = 5A$; $I_{10} = 3A$.



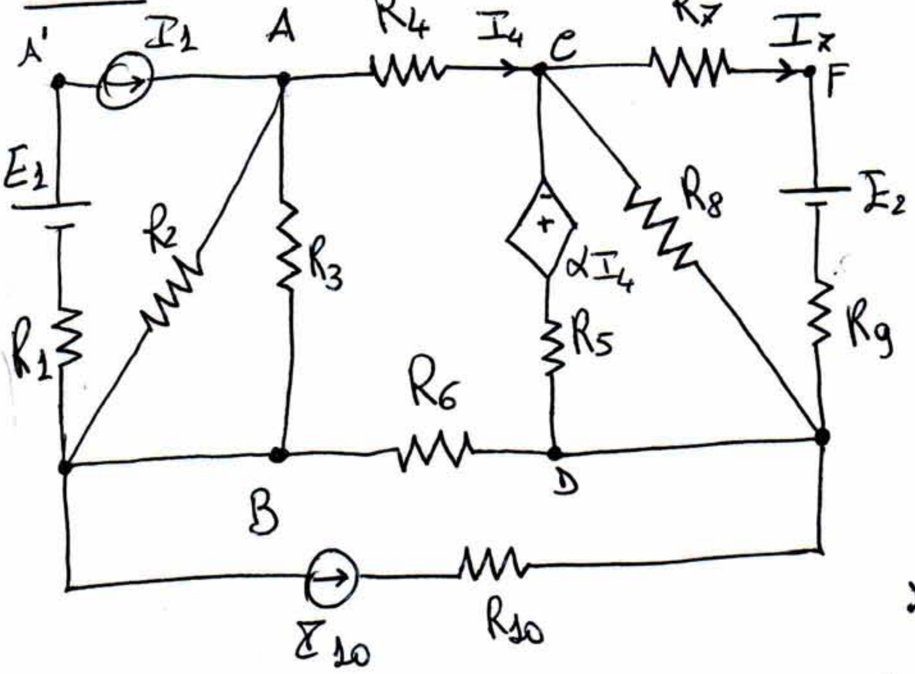
Esercizio 2:

Determinare il valore di C atta a rifasare totalmente il sistema.

$L = 10mH$; $\omega = 314$ rad/sec; $e(t) = 10\sqrt{2}\sin(\omega t + \pi/3)$ V; $R_1 = 5\ \Omega$; $R = 3\ \Omega$; $R_4 = 10\ \Omega$; $l_1 = 10cm$; $l_2 = 20cm$; $S = 10cm^2$; $N_1 = 20$; $N_2 = 30$; $N_3 = 40$; $\mu_r = 1000$.



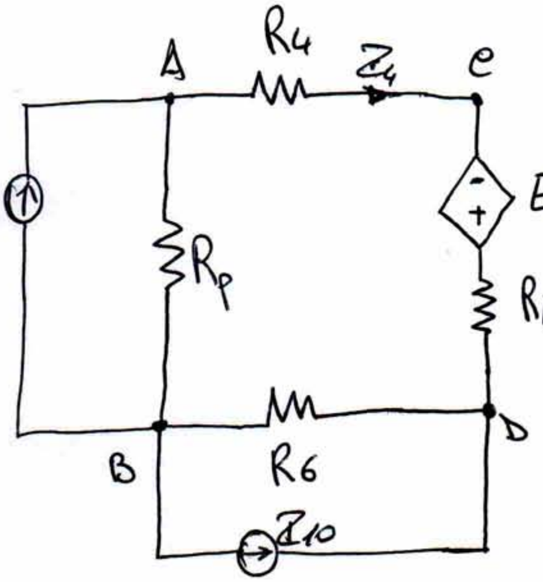
Ex. 1



$R_p = R_2 // R_3 = \frac{R_2 \cdot R_3}{R_2 + R_3} =$

$= \frac{2}{3}\,\Omega \cong 0{,}67\,\Omega$

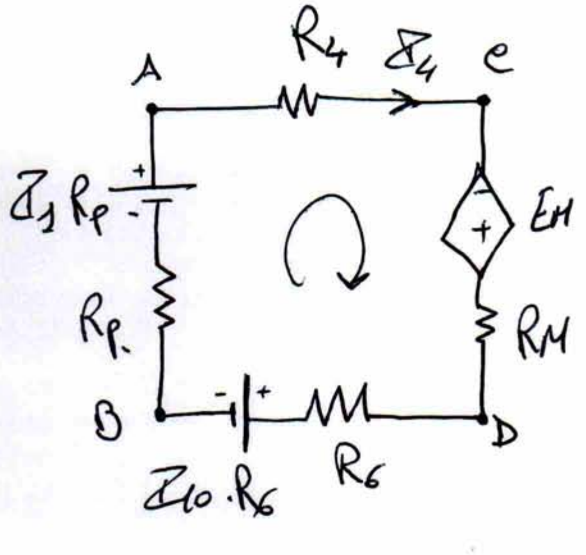
Ai fini della corrente, il generatore I_1 è prevalente rispetto ad E_1 ed R_1, per cui il circuito è possibile ridurlo come segue:



$$E_M = \frac{\frac{\alpha I_4}{R_5} - \frac{E_2}{R_7 + R_9}}{\frac{1}{R_5} + \frac{1}{R_8} + \frac{1}{R_7 + R_9}} =$$

$$= \frac{\frac{2}{3} I_4 - \frac{2}{12}}{\frac{1}{3} + \frac{1}{6} + \frac{1}{12}} = \frac{8 I_4 - 2}{7}$$

$R_M = 1{,}71\,\Omega$



Equazione alla maglia:

$$E_M - I_{10} R_6 + I_1 R_p = (R_4 + R_M + R_6 + R_p) I_4$$

$$\Rightarrow \frac{8}{7} I_4 - \frac{2}{7} - 12 + 3{,}35 = 11{,}38\, I_4 \Rightarrow$$

$$\Rightarrow I_4 = -0{,}87\,A$$

$$V_{AB} - I_1 R_p = -R_p I_4 \Rightarrow V_{AB} = I_1 R_p - R_p I_4 = R_p (I_1 - I_4) =$$

$$= 0{,}67\,(5 + 0{,}87) = 3{,}93\,V$$

La potenza generata da E_1 è: $P_{g_1} = E_1 I_1 = 25\,W$

La potenza erogata da E_1, considerando R_1 come la sua resistenza interna, è:

$$V_{A'B} - E_1 = -R_1 I_1 \Rightarrow V_{A'B} = E_1 - R_1 I_1 = 5 - 10 = -5\,V$$

$$P_{er_1} = V_{A'B} \cdot I_1 = -5 \cdot 5 = -25\,W$$

$$V_{CD} + E_M = R_M I_4 \Rightarrow E_M = \frac{8 I_4 - 2}{7} = -1{,}28\,V \Rightarrow$$

$$V_{CD} = -E_M + R_M I_4 = +1{,}28 - 1{,}49 = -0{,}21\,V$$

$$V_{CD} - E_2 = (R_7 + R_9) I_7 \Rightarrow I_7 = \frac{V_{CD} - E_2}{R_7 + R_9} = \frac{-0{,}21 - 2}{12} = -0{,}184\,A$$

La potenza generata da E_2 è:

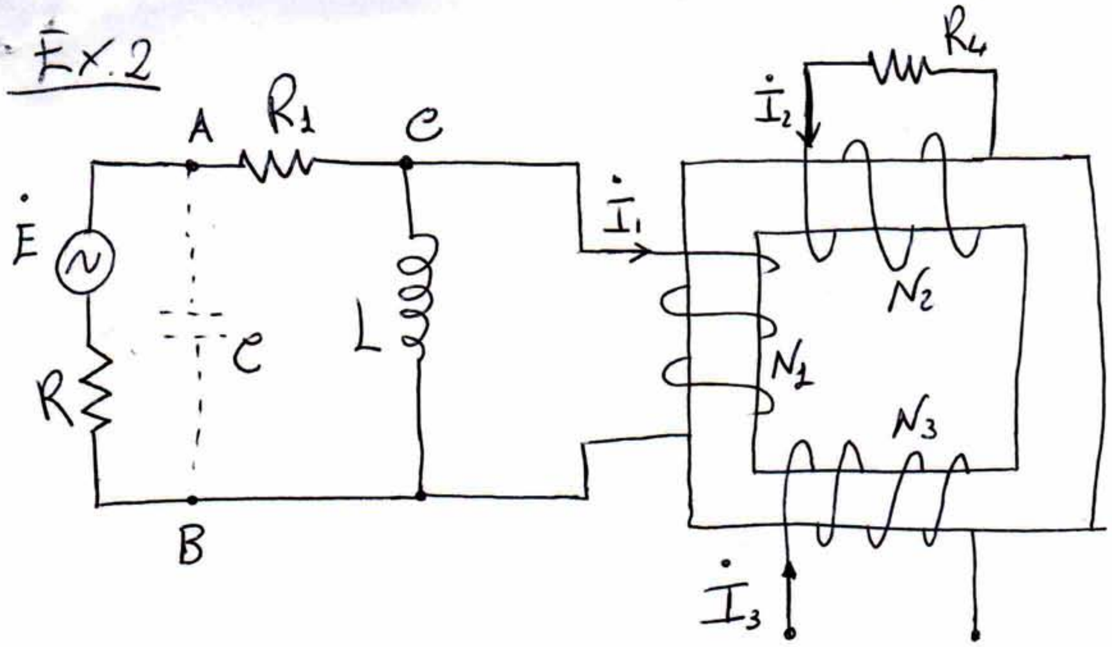
$$P_{g_2} = E_2(-I_7) = 0{,}37\,W$$

$$V_{FD} = E_2 + R_9 I_7 = 0{,}16\,V$$

La Potenza erogata da E_2 è:

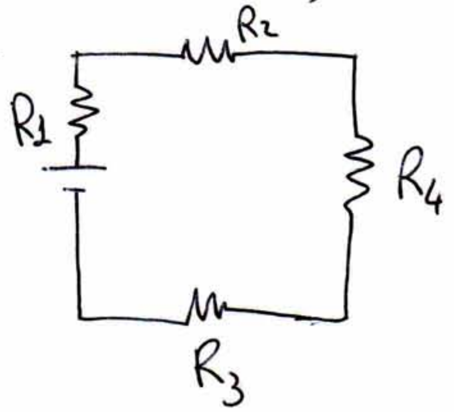
$$P_{er_2} = V_{FD}(-I_7) = 0{,}16 \cdot 0{,}184 = 29\,mW$$

Ex. 2



$$e(t) = 10\sqrt{2}\, sen\left(\omega t + \frac{\pi}{3}\right) \Rightarrow \dot{E} = 10\,(0{,}5 + j0{,}866)\ V$$

$$\mathcal{R}_1 = \mathcal{R}_4 = \frac{1}{\mu_0 \mu_r}\,\frac{l_1}{S} = 79{,}6\cdot 10^3\ \Omega^{-1}$$



$$\mathcal{R}_2 = \mathcal{R}_3 = \frac{1}{\mu_0 \mu_r}\cdot\frac{l_2}{S} = 159{,}2\cdot 10^3\ \Omega^{-1}$$

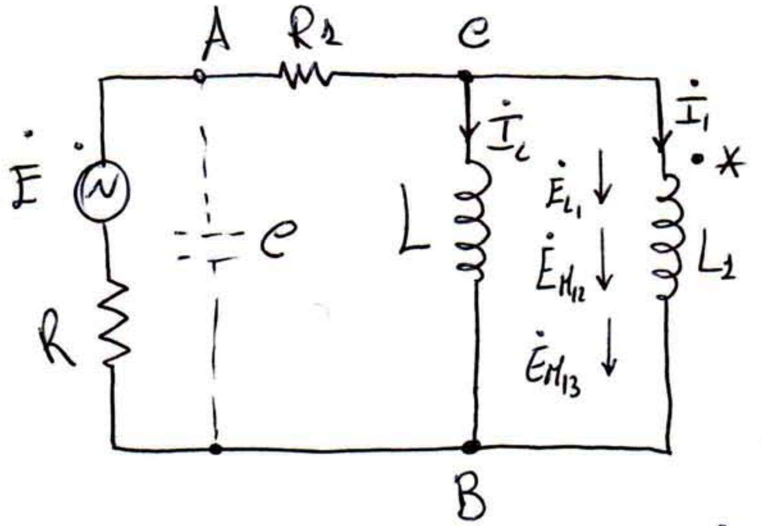
$$\mathcal{R}_{eq_1} = \mathcal{R}_{eq_2} = \mathcal{R}_{eq_3} = \mathcal{R}_{eq_4} = \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4 = 2\mathcal{R}_1 + 2\mathcal{R}_2 = 477{,}5\cdot 10^3\ \Omega^{-1}$$

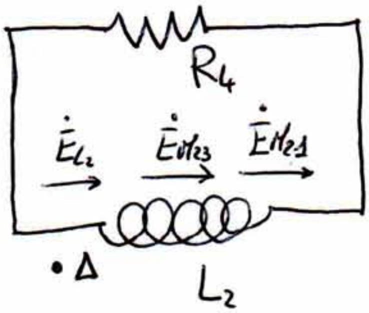
$$L_1 = \frac{N_1^2}{\mathcal{R}_{eq}} = 0{,}84\ mH\ ;\ L_2 = \frac{N_2^2}{\mathcal{R}_{eq}} = 1{,}88\ mH\ ;\ L_3 = \frac{N_3^2}{\mathcal{R}_{eq}} = 3{,}35\ mH$$

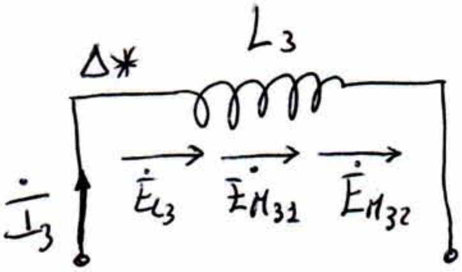
$$M_{12} = \frac{N_1 N_2}{\mathcal{R}_{eq}} = 1{,}26\cdot 10^{-3}\ H$$ (presa con segno positivo)

$$M_{13} = \frac{N_1 N_3}{\mathcal{R}_{eq}} = 1{,}67\cdot 10^{-3}\ H$$ (presa con segno positivo)

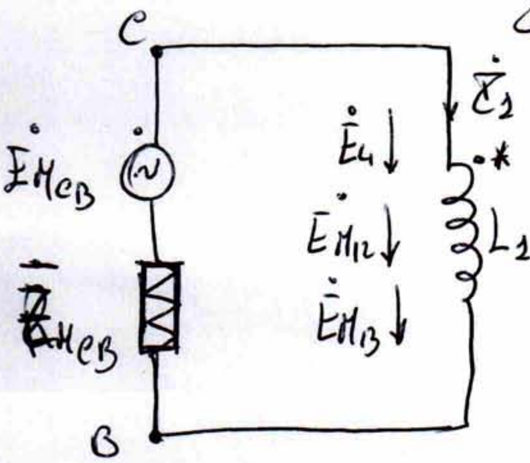
$$M_{23} = \frac{N_2 \cdot N_3}{\mathcal{R}_{eq}} = 2{,}5\cdot 10^{-3}\ H$$ (presa con segno positivo)

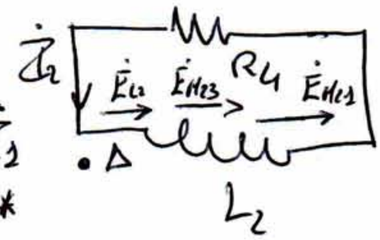




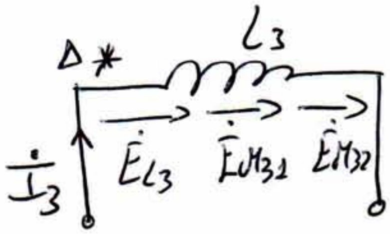


MILLMAN Tra C-L B e C-R1-Ė-R-B :





$$\dot{E}_{MCB} = \frac{\frac{\dot{E}}{R+R_1}}{\frac{1}{R_1+R}+\frac{1}{\bar{Z}_L}} \Rightarrow$$



$$\bar{Z}_L = j\omega L = 3{,}14j\ \Omega$$

$$\dot{E}_{MCB} = \frac{\dot{E}\,\bar{Z}_L}{R_1+R+\bar{Z}_L} = 2{,}85j - 2{,}28\ V$$

$$\bar{Z}_{MCB} = \frac{1}{\frac{1}{R+R_1}+\frac{1}{\bar{Z}_L}} = 2{,}72j + 1{,}07\ \Omega$$

Equazioni alle maglie:

$$\begin{cases} \dot{E}_{M_{CB}} + \dot{E}_{L_1} + \dot{E}_{M_{12}} + \dot{E}_{M_{13}} = \bar{Z}_{M_{CB}} \cdot \dot{I}_1 \\ \dot{E}_{L_2} + \dot{E}_{M_{21}} + \dot{E}_{M_{23}} = R_4 \dot{I}_2 \\ \dot{E}_{L_3} + \dot{E}_{M_{31}} + \dot{E}_{M_{32}} = 0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \dot{E}_{M_{CB}} - j\omega L_1 \dot{I}_1 - j\omega M_{12} \dot{I}_2 - j\omega M_{13} \dot{I}_3 = \bar{Z}_{M_{CB}} \cdot \dot{I}_1 \\ -j\omega L_2 \dot{I}_2 - j\omega M_{21} \dot{I}_1 - j\omega M_{23} \dot{I}_3 = R_4 \dot{I}_2 \\ -j\omega L_3 \dot{I}_3 - j\omega M_{31} \dot{I}_1 - j\omega M_{32} \dot{I}_2 = 0 \end{cases}$$

Essendo $\dot{I}_3 = 0$ (A ramo aperto la corrente è pari a zero)

$$\begin{cases} \dot{E}_{M_{CB}} - j\omega L_1 \dot{I}_1 - j\omega M_{12} \dot{I}_2 = \bar{Z}_{M_{CB}} \cdot \dot{I}_1 \\ -j\omega L_2 \dot{I}_2 - j\omega M_{21} \dot{I}_1 = R_4 \dot{I}_2 \\ -j\omega M_{31} \dot{I}_1 = j\omega M_{32} \dot{I}_2 \end{cases} \quad \ldots$$

$\ldots \quad \dot{I}_2 = -0{,}42 - 0{,}72j$ A ; $\dot{I}_1 = 0{,}62 + 1{,}07j$ A

$\dot{V}_{CB} - \dot{E}_{M_{CB}} = \bar{Z}_{M_{CB}} \cdot \dot{I}_1 \Rightarrow \dot{V}_{CB} = -4{,}53 + 5{,}67j$ V $\Rightarrow$

$\Rightarrow \dot{I}_L = \dfrac{\dot{V}_{CB}}{\bar{Z}_L} = 1{,}8 + 1{,}45j$ A ; $\dot{I} = \dot{I}_L + \dot{I}_1 = 2{,}42 + 2{,}52j$ A

La potenza complessa ($\mathcal{C}$) sarà:

$$\mathcal{C} = \dot{V}_{AB} \cdot \overset{\cup}{\dot{I}} = P + jQ \Rightarrow \dot{V}_{AB} - \dot{E} = -R\dot{I} \Rightarrow$$

$$\Rightarrow \dot{V}_{AB} = -2{,}26 + 1{,}1\,j \ V$$

$$\mathcal{C} = (-2{,}26 + 1{,}1\,j)(2{,}42 - 2{,}52\,j) = 2{,}69 + 7{,}44\,j \quad VAC$$

La potenza Reattiva atta a rifasare totalmente il sistema dovrà essere fornita in loco da un condensatore C, inserito tra i nodi A e B, la cui capacità sarà:

$$Q_C = Q = \omega C |\dot{V}_{AB}|^2 \Rightarrow C = \frac{Q}{\omega |\dot{V}_{AB}|^2} = 3{,}75 \ mF$$