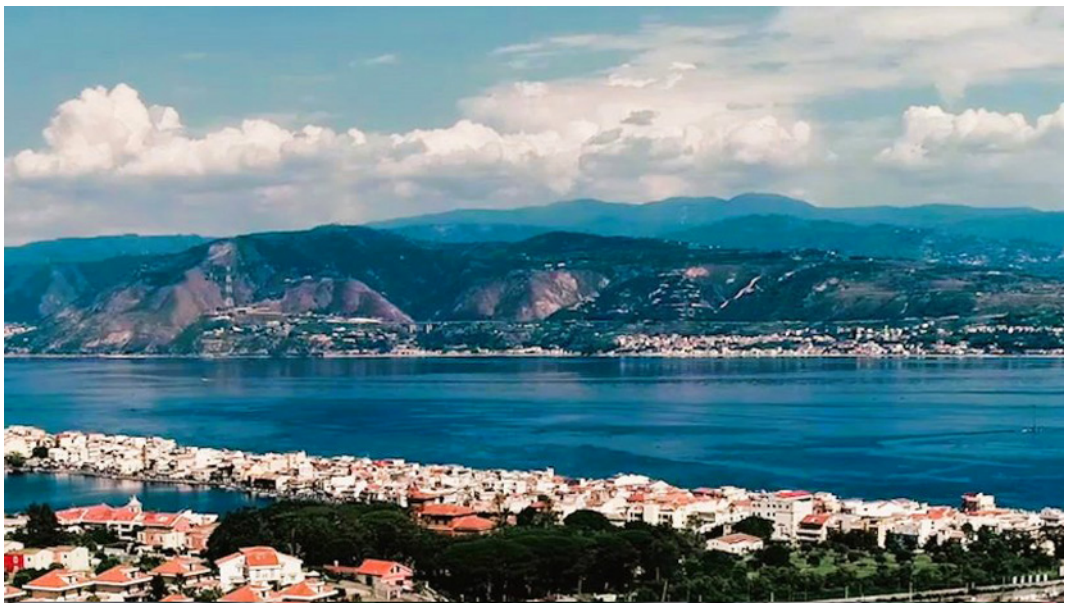

Book of Abstracts

Nonlinear elliptic equations in $\mathbb{R}^N$ involving the p-Laplacian operator

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In this talk, we discuss some existence and multiplicity results for nonlinear p-Laplacian Dirichlet problems defined in the whole space. In particular, the existence of two non-zero solutions with opposite energy sign for equations having combined effects of concave and convex nonlinearities is presented. The approach is based on variational methods.

References

[1] E. Amoroso, G. Bonanno, K. Perera, *Nonlinear elliptic p-Laplacian equations in the whole space* , Nonlinear Analysis **23** (2023), 113364

Lavrentiev's phenomenon for double phase functionals

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We shall consider double phase functionals of the form

$$\mathcal{F}[u] = \int_\Omega |\nabla u|^p + a(x)|\nabla u|^q \, dx \,, \quad 1 < p < q < \infty \,, \, 0 \le a(\cdot) \in L^\infty \,,$$

in the context of the Lavrentiev's phenomenon. That is, we want to exclude the situation in which minimizers of $\mathcal{F}$ cannot be appropriately approximated by regular functions, namely

$$\inf_{u_0 + W(\Omega)} \mathcal{F}[u] < \inf_{u_0 + C_c^\infty(\Omega)} \mathcal{F}[u] \,,$$

where $W(\Omega)$ is the energy space of the functional $\mathcal{F}$. To this purpose, one has to impose certain conditions on parameters p, q, and $a(\cdot)$. Many authors have considered a being Hölder continuous, with exponent controlled by q/p or $q - p$. The limitation of this approach is that p and q cannot be far from each other, as class $C^{0,\alpha}$ is trivial for $\alpha > 1$. In our recent work, we describe a new class of functions, allowing excluding the Lavrentiev's phenomenon for every choice of exponents p and q.

In the talk, we shall introduce double phase functionals, review the previous results concerning absence of Lavrentiev's phenomenon, and introduce our approach to the topic.

This is joint work with Iwona Chlebicka, Filomena de Filippis and Błażej Miasojedow.

References

[1] M. Borowski, I. Chlebicka, F. De Filippis, B. Miasojedow, *Absence and presence of Lavrentiev's phenomenon for double phase functionals upon every choice of exponents*, Calc. Var. Partial Differential Equations **63(2)** (2024), 1-23

Second-order regularity of solutions to nonlinear elliptic problems

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Second-order regularity results are established for solutions to elliptic equations and systems with principal part having Uhlenbeck structure and square-integrable right-hand sides. In the case of equations, differential operators depending on anisotropic norms of the gradient are also included. Both local and global estimates are obtained. The latter apply to solutions to homogeneous Dirichlet problems under minimal regularity assumptions on the boundary of the domain. In particular, no regularity of its boundary is needed if the domain is convex. A critical step in the approach is a sharp pointwise inequality for the involved elliptic operator. This talk is based on diverse joint investigations with C.A.Antonini, A.Kh.Balci, G.Ciraolo, L.Diening, A.Farina and V.Maz'ya.

Monotonicity formulas and (S_+)-property: old and new

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In this talk we discuss the relation between monotonicity formulas and the (S_+)-property, the fact that monotonicity formulas were originally developed for operators with a power-law growth like the p-Laplacian, and how the strategy of using monotonicity formulas to prove the (S_+)-property can be generalized to cases that grow similar to a power-law but are not exactly one (for example, a logarithmic perturbation of a power law).

Asymptotic proximity to higher order nonlinear differential equations

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Consider the Emden–Fowler type differential equation

$$u^{(n)} = r(t)|u|^{\lambda} \operatorname{sgn} u, \quad \lambda > 0,\ \lambda \neq 1, \tag{1}$$

where r is a continuous function for $t \geq 1$. The problem of the proximity of solutions of (1) with solutions of the unperturbed equation $y^{(n)} = 0$ has a long history, started over seventy years ago by Kiguradze in [5], motivated by Sobol [7]. Recently, this study continued, by considering the higher order differential equation

$$u^{(n)} + q(t)u^{(n-2)} + r(t)|u|^{\lambda} \operatorname{sgn} u = 0, \quad n \geq 3, \tag{2}$$

where q is a positive continuous function for $t \geq 1$ and giving conditions for the existence of solutions of (2) which are close, for large t, to ones of the linear equation $y^{(n)} + q(t)y^{(n-2)} = 0$. Interesting contributions can be found in [1, 2, 3, 4, 6, 8]. In these studies, two cases are distinguished, according to the second order differential equation $h''(t)+q(t)h(t) = 0$ is oscillatory or nonoscillatory. In the first case, an asymptotic representations for oscillatory solutions is given and, in the second one, the existence of solutions with polynomial-like or noninteger power-law asymptotic behavior is obtained.

In final part of the lecture some results from [2] are presented. In this case, equation (2) can be written as a suitable two-term equation and the study is accomplished by using a general topological approach, based on an induction method and an iterative process, see [3]. Some suggestions for further research complete the talk.

References

[1] Astashova I.V, Springer Proc. Math. Stat. **164** (2016), 191–203.

[2] Astashova I., Bartušek M., Došlá Z., and Marini, Adv. Nonlin. Anal. **11** (2022), 1598–1613.

[3] Bartušek M., Cecchi M., Došlá Z., Marini M., J. Math. Anal. Appl. **388** (2012), 1130–1140.

[4] Došlá Z., Liška P., Marini M., J. Fixed Point Theory Appl. **21** (2019), 1-16.

[5] Kiguradze I.T., Sb. Math. **65** (1964), 172–187 (Russian).

[6] Otrocol D., Rend. Circ. Mat. Palermo **2-65** (2016), 387-393.

[7] Sobol I. M., Dokl. Akad. Nauk. **61** (1948), 219–222 (Russian).

[8] Zhou Z., Yan Y., J. Appl. Anal. Comput. **8** (2018), 549-557.

On the existence of solutions for second order differential inclusions

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The talk will be about new results, obtained in recent joint works with Tiziana Cardinali ([4], [5], [6]), on the existence of mild solutions for the following nonlocal problem driven by a semilinear second order differential inclusion

$$\textbf{(P)} \quad \begin{cases} x''(t) \in A(t)x(t) + F(t, N(t)x) \\ x(0) = g(x) \\ x'(0) = h(x) \end{cases}$$

and for an initial problem governed by a Sturm-Liouville differential inclusion

$$\textbf{(SL)} \quad \begin{cases} (p(t)x'(t)) \in \lambda G(t)x(t) + F(t, x(t)) \\ x(0) = x_0 \\ x'(0) = \overline{x}_0. \end{cases}$$

The study of nonlocal problems has been well recognized as a fruitful area in mathematics, in particular the nonlocal initial conditions are more appropriate then the classical one to describe natural phenomena because it allows us to consider additional information. Therefore these problems have been widely studied because of their applications (see [2], [7] and the reference cited therein). Moreover, it is also well-known the importance of Sturm-Liouville theory: many differential equations can be trasformed in the Sturm-Liouville ones and the study of differential equations or inclusions involving Sturm-Liouville operators is stimulated by problems related to various area of applied sciences.
In this talk we discuss about the existence of mild solutions for **(P)** and **(SL)** in Banach spaces in the lack of compactness both on the fundamental system generated by the linear part and on the nonlinear multivalued term. For lack of compactness we have that our theorems extends in a broad sense the results presented in [3] and [8]. For **(SL)** we are able to establish the existence of strong solutions too. The method used for proving the existence of mild solutions is based on the combination of a fixed point theorem and a selection theorem developed by ourselves. Moreover we use De Blasi measure of noncompactness and the weak topology. This approach is present in [1], but with the aim of studying the existence of mild solutions for problems controlled by semilinear first order differential inclusions. As application of our results for **(P)** we study of the controllability of a Cauchy problem guided by the following wave equation

$$\frac{\partial^2 w}{\partial t^2}(t,\xi) = \frac{\partial^2 w}{\partial \xi^2}(t,\xi) + b(t)\frac{\partial w}{\partial \xi}(t,\xi) + T(t)w(t,\cdot)(\xi) + u(t,\xi),$$

while we apply the existence theorem proved for **(SL)** to analyse the controllability of a Cauchy problem driven by

$$((t^2+1)x'(t))' = \lambda \sin(t)x(t) + h(t) + u(t)$$

References

[1] I. Benedetti, M. Väth, Semilinear inclusion with nonlocal conditions without compactness in non-reflexive spaces, *Topol. Methods Nonlinear Anal.* **48**(2016), 613–636.

[2] Byszewski L., Theorems about the existence and uniqueness of solutions of a semilinear evolution nonlocal Cauchy problem. J. Math. Anal. Appl. **162** No. 2 (1991), 494-505.

[3] Cardinali T., De Angelis E., Non-autonomous second order differential inclusions subject to a perturbation with a stabilizing effect. Results Math. **78** No. 8 (2021).

[4] Cardinali T., Duricchi G., On nonlocal problems for semilinear second order differential inclusions without compactness. Electron. J. Qual. Theory Differ. Equ. No. 66 (2021), 1-32.

[5] Cardinali T., Duricchi G., Further study on second order nonlocal problems monitored by an operator: an approach without compactness. Electron. J. Qual. Theory Differ. Equ. No. 13 (2023), 1-34.

[6] Cardinali T., Duricchi G., Strong solutions and mild solutions for Sturm-Liouville differential inclusions. Submitted

[7] K. Deng, Exponential Decay of Solutions of Semilinear Parabolic Equations with Nonlocal Initial Conditions, *J. Math. Anal. Appl.* **21**(1993), 630–637.

[8] H. R. Henríquez, V. Poblete, J. C. Pozo, Mild solutions of nonautonomous second order problems with nonlocal initial conditions, *J. Math. Anal. Appl.* **412**(2014), 1064–1083.

Existence and multiplicity of positive solutions for a p-Laplacian equation with sub-critical singular parametric reaction term

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We consider the following singular elliptic problem

$$\begin{cases} -\Delta_p u = \lambda f(x,u) + \mu g(x,u) & \text{in} \quad \Omega, \\ u > 0 & \text{in} \quad \Omega, \\ u = 0 & \text{on} \ \partial\Omega, \end{cases} \tag{3}$$

where $\Omega \subseteq \mathbb{R}^N$ $(N \geq 3)$ is a bounded domain with smooth boundary $\partial\Omega$, $p \in (1,N)$, λ, μ are real positive parameters, $\Delta_p u := \mathrm{div}(|\nabla u|^{p-2}\nabla u)$ is the classical p-Laplacian, $f : \Omega \times (0,+\infty) \to [0,+\infty)$ and $g : \Omega \times [0,+\infty) \to [0,+\infty)$ are Carathéodory functions satisfying the following assumptions:

(Q_1) $\lim\limits_{s \to 0^+} f(x,s) = +\infty$ uniformly w.r.t. $x \in \Omega$,

(Q_2) $f(x,s) \leq c_1 s^{-\gamma} + c_2$ for a.a. $x \in \Omega$, $\forall s > 0$, $\gamma \in (0,1)$,

(Q_3) $g(x,s) \leq c_3 s^{q-1} + c_4$ for a.a. $x \in \Omega$, $\forall s \geq 0$, $q \in (1,p^*)$,

where $p^* = \frac{Np}{N-p}$ is the critical Sobolev exponent and c_i for $i \in \{1,...,4\}$ are non-negative constants. Combining variational with truncation techniques, a quantitative estimate of the parameters λ, μ, for which problem (3) admits a positive solution, is obtained. In particular, this solution is a local minimum of a suitable functional. Furthermore, if $p < q < p^*$, and, in addition, g fulfills a classical unilateral Ambrosetti-Rabinowitz condition, the Mountain Pass Theorem ensures the existence of a second positive solution.

References

[1] P. Candito, G. Failla, R. Livrea, *Positive solutions for a p-Laplacian equation with sub-critical singular parametric reaction term*, submitted

Symmetrization results for general nonlocal linear elliptic and parabolic problems

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We discuss a Talenti-type symmetrization result in the form of mass concentration (*i.e.* integral comparison) for very general linear nonlocal elliptic problems, equipped with homogeneous Dirichlet boundary conditions. In this framework, the relevant concentration comparison for the classical fractional Laplacian can be reviewed as a special case of our main result, thus generalizing previous results obtained in collaboration with B. Volzone. Also a Cauchy-Dirichlet nonlocal linear parabolic problem is considered. The results are contained in a joint paper with G. Piscitelli and B. Volzone.

Strongly singular convective elliptic equations in $\mathbb{R}^N$ driven by a non-homogeneous operator

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In this communication, concerning a work with U. Guarnotta, I present an existence result for strongly singular convective elliptic equation driven by a non-homogeneous differential operator, patterned after the (p, q)-Laplacian, in the whole space. This problem exhibits at least four interesting peculiarities: (i) the operator is non-homogeneous, (ii) the reaction term is convective: this destroys the variational structure of the problem, (iii) the reaction term is strongly singular, and (iv) the problem is set in the whole space: this causes lack of compactness of Sobolev embeddings, and compels to use some localization procedures. Thus, to prove the existence result, we solve some regularized and localized problems through fixed point theory, variational methods, truncation techniques, and a sub-super-solution theorem. Then, we pass to the limit employing an energy estimate obtained by a localization procedure. Finally, using a compactness argument, we gain a solution to the problem.

Birkhoff-Kellogg type results with applications

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We present some classical and recent results of Birkhoff-Kellogg type in cones and wedges. We illustrate their applicability in the context of ordinary, functional and partial differential differential equations subject to local, nonlocal and functional boundary conditions.

References

[1] G. D. Birkhoff and O. D. Kellogg, Invariant points in function space, *Trans. Amer. Math. Soc.*, **23** (1922), 96–115

[2] A. Calamai and G. Infante, An affine Birkhoff–Kellogg type result in cones with applications to functional differential equations, *Mathematical Methods in the Applied Sciences*, **46** (2023), 11897–11905

[3] A. Calamai and G. Infante, On fourth order retarded equations with functional boundary conditions: a unified approach, *Discrete and Continuous Dynamical Systems - Series S*, doi: 10.3934/dcdss.2023006

[4] A. Calamai and G. Infante,Nontrivial solutions of a parameter-dependent heat flow problem with deviated arguments, to appear in: "Topological Methods for Delay and Ordinary Differential Equations", P. Amster and P. Benevieri (Eds.)

[5] A. Calamai, G. Infante and Jorge Rodríguez–López, A Birkhoff–Kellogg type theorem for discontinous operators with applications, submitted, arXiv:2401.16050

[6] G. Infante, Eigenvalues of elliptic functional differential systems via a Birkhoff-Kellogg type theorem, *Mathematics*, **9** (2021), n. 4

[7] G. Infante, On the solvability of a parameter-dependent cantilever-type BVP, *Appl. Math. Lett.*, **132** (2022), 108090

[8] M. A. Krasnosel'skiĭ and L. A. Ladyženskiĭ, The structure of the spectrum of positive nonhomogeneous operators, *Trudy Moskov. Mat. Obšč*, **3** (1954), 321–346

Minimal energy solutions to potential singular ϕ-Laplacian systems

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In this talk we discuss the solvability of a nonlinear system involving a singular ϕ-Laplacian operator

$$u \mapsto \left[\phi(u')\right]'$$

associated with a potential multivalued boundary condition expressed in terms of subdifferential of a convex function. Here, ϕ is a homeomorphism from an open ball of radius a, centered at the origin B_a onto $\mathbb{R}^N$such that $\phi(0_{\mathbb{R}^N}) = 0_{\mathbb{R}^N}$, $\phi = \nabla\Phi$, with $\Phi : \overline{B}_a \to (-\infty, 0]$ of class C^1 on B_a, continuous and strictly convex on $\overline{B}_a$. First, we provide a variational approach in the frame of critical point theory for convex, lower semicontinuous perturbations of C^1-functionals. Then we derive the existence of solutions by minimizing the corresponding energy functional. Among other often invoked features in ensuring the existence of minimizers, the impact of the boundary condition in relation with the singular character of the operator is emphasized.

Boundary value problems on the half-line for (generalized) curvature operator and Kneser solutions

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Second order nonlinear equations of the form

$$(a(t)\Phi(x'))' + F(t,x) = 0, \quad t \geq t_0 \tag{4}$$

are considered, where the functions a and F are continuous, with a positive, and the operator Φ can be the (generalized) one-dimensional curvature operator

$$\Phi(u) = |u|^\alpha (1-|u|^{1+\alpha})^{\frac{-\alpha}{1+\alpha}} \operatorname{sgn} u, \quad \alpha > 0,$$

or, more generally, an increasing odd homeomorphism. We analyze the problem of existence of globally positive solutions satisfying some boundary conditions related to the existence of radial solutions for PDEs with operators in divergence form. Boundary value problems associated to (4) or, more generally, to partial differential equations whose radial solutions satisfy (4), have been investigated by many authors, see for instance [1, 2, 3] and references therein.

In the second part of the talk, we present some results for problems related to the so-called Kneser solutions and intermediate solutions for (4). Our method is based on a new abstract fixed point result for invertible operators and on asymptotic properties of auxiliary half-linear differential equations, and give some "similarity" results between (4) and suitable half-linear equations. In particular, the concept of principal solutions and some comparison results play a fundamental role in finding good a-priori bounds for solutions.

References

[1] A. Azzollini, *Ground state solutions for the Hénon prescribed mean curvature equation*, Adv. Nonlinear Anal. **8** (2019), 1227-1234.

[2] G. Feltrin, M. Garrione, *Homoclinic and heteroclinic solutions for non-autonomous Minkowski-curvature equations*, Nonlinear Anal., Theory Methods Appl., Ser. A, **239** (2024), Article ID 113419, 21 p.

[3] P. Jebelean, J. Mawhin, C. Şerban, *A vector p-Laplacian type approach to multiple periodic solutions for the p-relativistic operator*, Commun. Contemp. Math. **19** (2017), 1-16.

[4] Z. Došlá, M. Marini, S. Matucci, *Kneser solutions for equations with the generalized relativistic operator*, preprint.

[5] Z. Došlá, S. Matucci, *Ground state solutions to nonlinear equations with p-Laplacian*, Nonlinear Anal., Theory Methods Appl., Ser. A, **184** (2019), 1–16.

Nonlinear Sturm Liouville problems with sign changing coefficients

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In this talk we present an existence result of solutions for a complete parametric Sturm-Liouville equation under Dirichlet boundary condition with sign changing coefficients. Under specific hypotheses on the nonlinear term, through a critical point theorem, we obtain two nontrivial and nonnegative solutions with opposite energy sign.

References

[1] E. Amoroso, G. D'Aguì, V.Morabito, *On a complete parametric Sturm-Liouville problem with sign changing coefficients,* ,[J]. AIMS Mathematics, 2024, 9(3): 6499-6512

Population dynamics models with seasonality

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In this talk we will consider predator-prey and competition models. We will give an overview of recent results obtained for the case when seasonality is considered.

We will describe results on extinction, persistence, coexistence and of stability of the coexistence orbit.

The results obtained mostly using topological methods are described in the papers [1,2,3,4].

References

[1] Coelho, I., Rebelo, C., Sovrano, E., Extinction or coexistence in periodic Kolmogorov systems of competitive type, *Discrete Contin. Dyn. Syst.*, 41, 2021, pp. 5743-5764

[2] Garrione, M., Rebelo, C., Persistence in seasonally varying predator-prey systems via the basic reproduction number, *Nonlinear Anal. Real World Appl.*, 30, 2016, pp. 73-98

[3] Ortega, V., Rebelo, C., A note on stability criteria in the periodic Lotka-Volterra predator-prey model Ortega, *Appl. Math. Lett.*, 145, 2023, Paper No. 108739, 8 pp.

[4] Rebelo, C., Soresina, C., Coexistence in seasonally varying predator-prey systems with Allee effect, *Nonlinear Anal. Real World Appl.*, 55, 2020, 103140, 21 pp.

Multiplicity for Dirichlet systems with discrete relativistic operator

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We discuss the solvability of some Dirichlet systems involving the relativistic discrete operator

$$u \mapsto \Delta \left[\frac{\Delta u(n-1)}{\sqrt{1-|\Delta u(n-1)|^2}} \right] \qquad (n \in \{1, \dots, T\}) .$$

Here, for $u : \{0, \dots, T+1\} \to \mathbb{R}^N$, we denote by Δu the usual forward difference operator. Besides an "universal" existence result for a system with a general nonlinearity, we obtain multiplicity of solutions for potential systems with parameterized nonlinearities. Our approach relies on Brouwer degree arguments and critical point theory for convex, lower semicontinuous perturbations of C^1-functionals. Comparisons with similar results for the relativistic differential operator and for the mean curvature operator in Minkowski space are also provided.

Based on joint work with Alberto Cabada and Petru Jebelean.

Multiplicity of solutions for a periodic boundary Sturm-Liouville problem with discontinuous reaction term

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We investigate, via variational methods, the following periodic boundary value problem with the Sturm-Liouville equation

$$\begin{cases} -(pu')' + qu = \lambda f(x, u(x)) & \text{in }]0,T[, \\ u(0) = u(T),\ u'(0) = u'(T), \end{cases} \tag{P_λ}$$

where $p, q \in L^\infty([0,T])$ satisfying $p(0) = p(T)$, $q_0 \coloneqq \operatorname{essinf}_{[0,T]} q > 0$, $p_0 \coloneqq \operatorname{essinf}_{[0,T]} p > 0$ and the reaction term $f : [0,T] \times \mathbb{R} \to \mathbb{R}$ belongs to a suitable set of almost everywhere continuous functions called highly discontinuous. In particular the set of the points of discontinuity of f may also be uncountable. Under proper growth conditions on f, the existence of three weak solutions to Problem (P_λ) is established, for an appropriate range of the parameter λ, using the critical point theory for non-differentiable functions.

References

[1] Livrea R., Vassallo B., *Three weak solutions to a periodic boundary Sturm-Liouville problem with discontinuous reaction*, preprint, (2023).